Accounting for refractive index effects in multilateration

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Outline

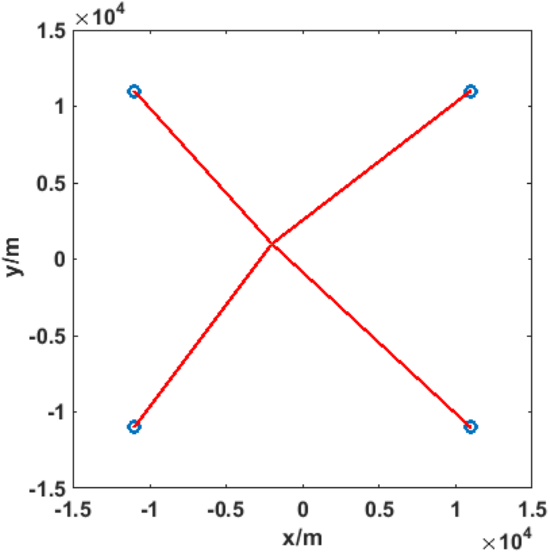
laser interferometry

- laser sends a beam to a reflecting target (e.g.,corner cube mirror)
- reflected beam interferes with the outgoing beam
- interference fringe counts track the changes in the *optical* distance from the station $\boldsymbol{p}$ to the target $\boldsymbol{x}$
- want to determine the *geometric* distance $\|\boldsymbol{x} - \boldsymbol{p}\|$
- refractive index field $n(\boldsymbol{x})$, observed optical distance D:

$$D = \|\boldsymbol{x}-\boldsymbol{p}\| n(\boldsymbol{x}, \boldsymbol{p}), \quad n(\boldsymbol{x}, \boldsymbol{p}) = \frac{1}{\|\boldsymbol{x} - \boldsymbol{p}\|} \int_0^1 n(\boldsymbol{p}+s(\boldsymbol{x}-\boldsymbol{p}))\mathrm{d}s$$

- statistical model for $n(\boldsymbol{x})$ determines $p(D|\boldsymbol{x}, \boldsymbol{p})$

Multilateration



Usual statistical model for length measurements

- Stations located at $\boldsymbol{p}_k$ measuring distances to targets $\boldsymbol{x}_j$:

$$d_i = \|\boldsymbol{x}_k - \boldsymbol{p}_k\|(1 + \delta_i) + \epsilon_i, \quad \delta_i \in \mathrm{N}(0, \sigma_R^2), \quad \epsilon_i \in \mathrm{N}(0, \sigma_A^2),$$

- Nonlinear least squares bundle adjustment

$$\min_{\{\boldsymbol{x}_j\},\{\boldsymbol{p}_k\}} \sum_i w_i^2 (d_i - \|\boldsymbol{x}_j - \boldsymbol{p}_k\|)^2, \quad w_i^2 = \frac{1}{\sigma_A^2 + d_i^2 \sigma_R^2}$$

- δ_i and ϵ_i assumed to be statistically independent over i
- But refractive index along nearby paths at nearby times likely to be similar

Model of refractive index of air

- Refractive index is a function of temperature, pressure, humidity, wavelength, CO_2 content
- An increase of 1 ^{0}C corresponds to a decrease of approximately 1 part in 10^6 in refractive index
- Spatio-temporal model for refractive index $n(\boldsymbol{x}, t)$

$$\mathrm{cov}(n, n') = k(\|\boldsymbol{x} - \boldsymbol{x}'\|)k(|t - t'|)$$

- If

$$d_i = \|\boldsymbol{x}_j - \boldsymbol{p}_k\| n_i + \epsilon_i,$$

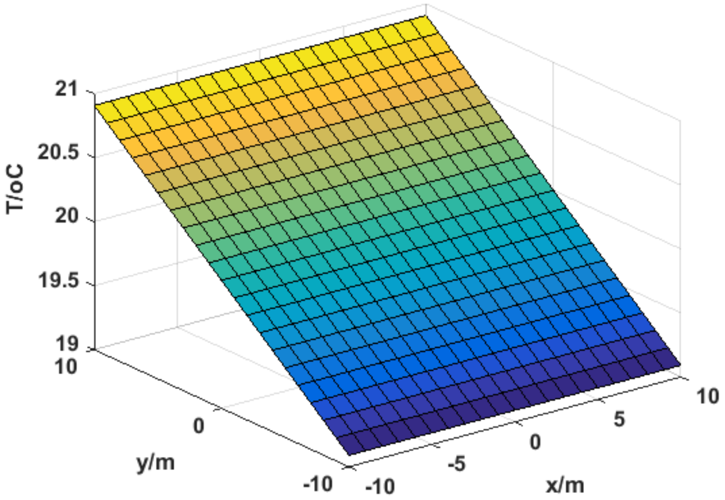
we can determine

$$n_i \sim \mathrm{N}(\hat{n}_i, \sigma_i^2),$$

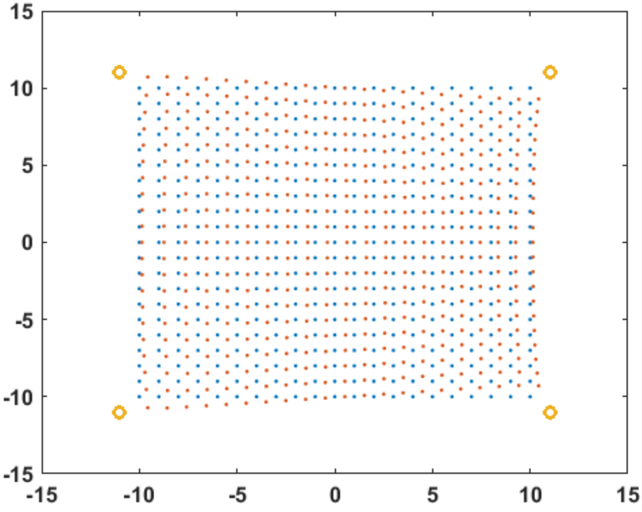
and hence

$$\boldsymbol{d}|\{\boldsymbol{x}_j, \boldsymbol{p}_k\} \sim \mathrm{N}(\hat{\boldsymbol{d}}, V_{\boldsymbol{d}})$$

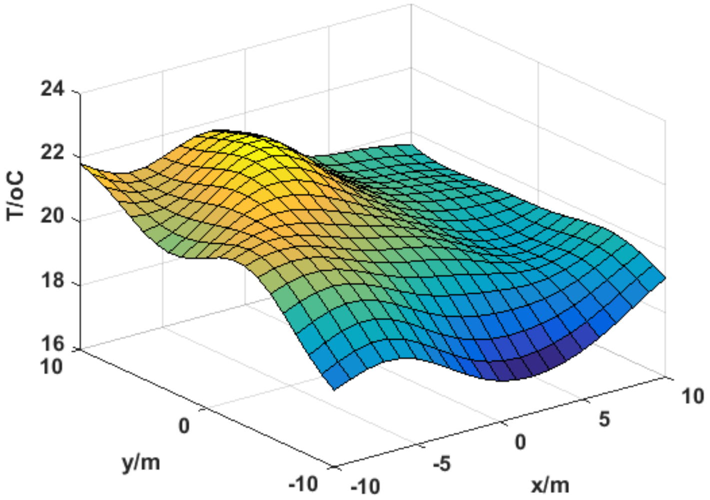
Temperature as a linear function of location



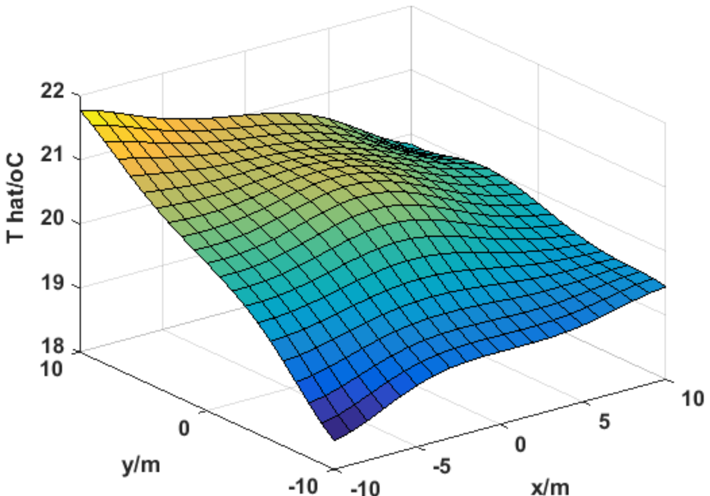
Target estimates, constant refractive index assumption



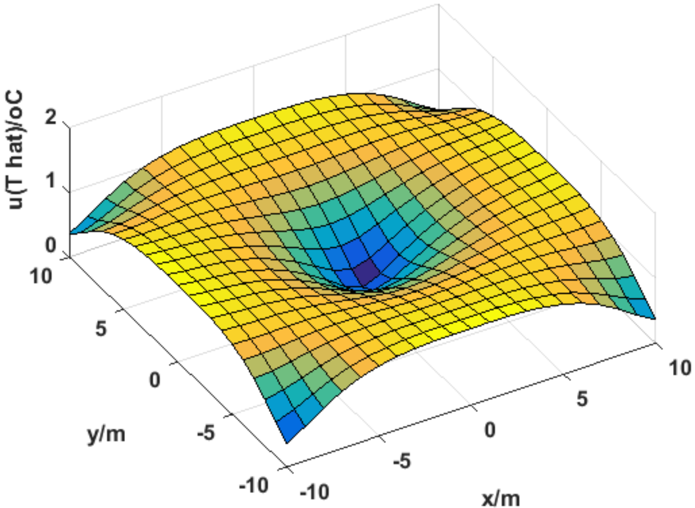
Temperature as a GP function of location



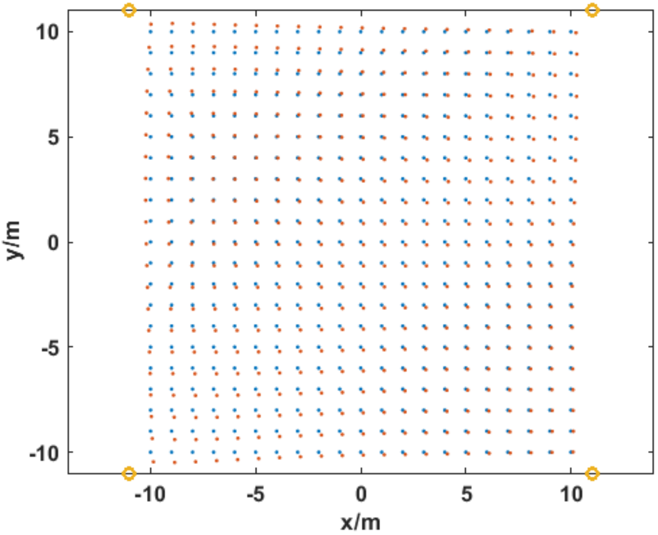
Estimated temperature field based on 5 temperature measurements



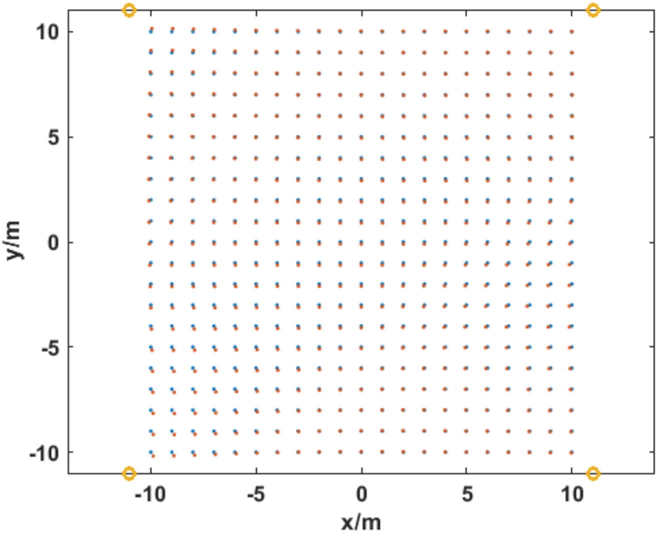
Uncertainty associated with temperature estimates



Target estimates, constant refractive index assumption



Target estimates, estimated refractive index



Robotic assembly model

- Targets $\boldsymbol{y}_{j,0}$ attached to the robot's actuator head
- Moves as rigid body, location of the targets in the lth position

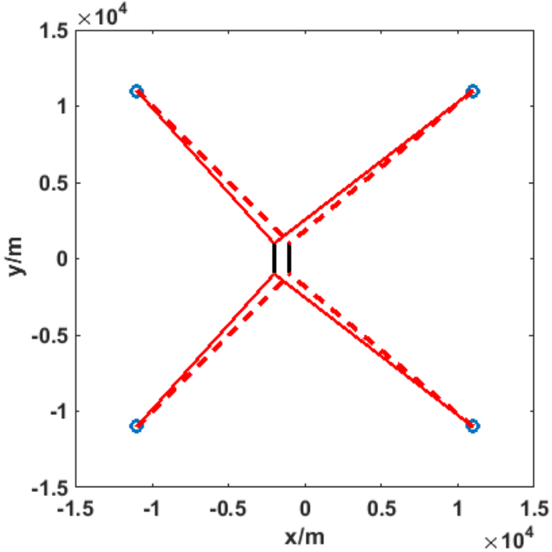
$$\boldsymbol{y}_{j,l} = R^{\mathrm{T}}(\alpha_l)\boldsymbol{y}_{j,0} + \boldsymbol{x}_{0,l}$$

- Interferometric measurements

$$d_i = \|\boldsymbol{y}_{j,l} - \boldsymbol{p}_k\| n_i + \epsilon_i$$

- Determine best estimates of α_l and $\boldsymbol{x}_{0,l}$

Robotic assembly



Uncertainties associated with robotic assembly

Location/micrometres

6.3	6.1	5.9	5.9	5.9	6.1	6.3
8.8	9.0	9.1	9.1	9.1	9.0	8.8
9.0	8.9	8.8	8.7	8.8	8.9	9.0
8.8	9.0	9.1	9.1	9.1	9.0	8.8
10.0	9.8	9.7	9.6	9.7	9.8	10.0
9.4	9.6	9.8	9.8	9.8	9.6	9.4

Uncertainties associated with robotic assembly

Change in location/micrometres

2.2	2.6	3.2	3.9	4.5	5.1
2.1	2.3	2.5	2.8	3.1	3.5
12.6	12.6	12.6	12.6	12.6	12.7
12.5	12.6	12.6	12.6	12.5	12.4
14.0	13.9	13.9	13.9	14.0	14.1
13.5	13.6	13.6	13.6	13.5	13.4

Summary

- Multilateration used in large scale dimensional metrology
- Refractive index a limiting factor
- More realistic models for refractive index effects
    - Better estimates of target locations
    - More realistic uncertainty evaluation
    - Better experimental design

Selected references

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